> List Notes.

1. Definitions: Vectors., Velocities. (Tangential Vectors), Normal Vectors, Orthogonal Vectors, Basis, etc.

2. Operations of Vectors.:
   Addition. Scalar Multiplication. Dot (inner) Product, Cross Product (3D). [Magnitute & Direction].
   Norms. ( $\underbrace{2\|\vec{x}\|^2 + 2\|\vec{y}\|^2 = \|\vec{x}+\vec{y}\|^2} + \|\vec{x}-\vec{y}\|^2$ )

   Remark: About this. , in 1-D.

$$(x+y)^2 + (x-y)^2 = 2x^2 + 2y^2.$$
$$(x+y)^2 - (x-y)^2 = 4x \cdot y.$$

3. Projection.
4. Determinant of Square $(n \times n)$ Matrix. What it means? and How to use it?

5. Lines. Planes.
   a> Descriptions (Parametrization, Equations, etc)
      In Particular, Intersection of two Planes.
   b>. Distances

| | | |
|---|---|---|
| Line | ~~and~~ to | line, (Optional). |
| Point | to | Line |
| Point | to | Plane |
| Line | to | Plane |
| Plane | to | Plane. |

   under Different ~~Des~~criptions.

6). Quadratic form. How to normalize. Quadratic forms?
   $[Ax^2 + By^2 + Cz^2 = D.]$

   Hint: Using Matrix

7. Curves. Differentiation Rule. (w.r.t. Parametrization) [Basically 1-D Calculas].
8. Arc Length. Curvature, Normal Vectors, Decomposition.
9. Domain of Functions of Multiple-variables & Level Set.

> Spring & Pendulum (Application)
弹簧  钟摆.

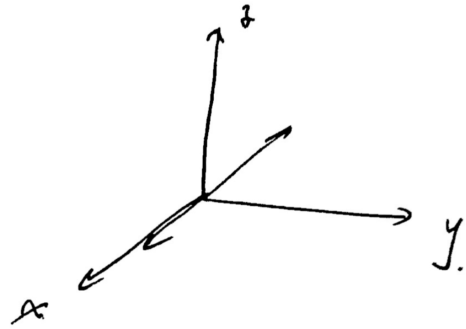
$$\frac{d^2\vec{r}}{dt^2} = -\vec{r}$$

a> Hooke's Law
b> Newton's Second Law

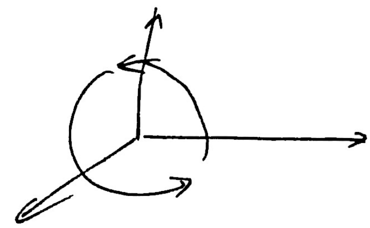
$\vec{r} = (r_1, r_2, r_3)$

Two Typical cases

a>



b>



$\vec{r}(0) = \vec{r}_0$  $\vec{r}'(0) = \frac{d}{dt}\vec{r}\Big|_{t=0} = \vec{r}_0'$

Let $\vec{v} = \vec{r}'$

Then $\frac{d}{dt}\begin{pmatrix}\vec{r}\\ \vec{v}\end{pmatrix} = \begin{pmatrix}\vec{v}\\ -\vec{r}\end{pmatrix} = \begin{pmatrix}0 & I_3\\ -I_3 & 0\end{pmatrix}\begin{pmatrix}\vec{r}\\ \vec{v}\end{pmatrix}$

Characteristics a> $\begin{pmatrix}\vec{r}\\ \vec{v}\end{pmatrix}(t) = \begin{pmatrix}\vec{d}_1 \sin t\\ \vec{d}_1 \cos t\end{pmatrix}$  initial $\begin{pmatrix}0\\ \vec{d}_1\end{pmatrix} = \begin{pmatrix}\vec{r}_0\\ \vec{v}_0\end{pmatrix}$.

$\forall\ \vec{d}_1, \vec{d}_2 \in \mathbb{R}^3$

b> $\begin{pmatrix}\vec{r}\\ \vec{v}\end{pmatrix}(t) = \begin{pmatrix}+\vec{d}_2 \cos t\\ -\vec{d}_2 \sin t\end{pmatrix}$  $\begin{pmatrix}\vec{r}_0\\ \vec{v}_0\end{pmatrix} = \begin{pmatrix}+\vec{d}_2\\ 0\end{pmatrix}$

Important Thing: $\begin{pmatrix}0\\ \vec{d}_1\end{pmatrix}$, $\begin{pmatrix}-\vec{d}_2\\ 0\end{pmatrix}$ linear independent.

Then Any initial Data $\begin{pmatrix}\vec{r}_0\\ \vec{v}_0\end{pmatrix} = \begin{pmatrix}\vec{r}_0\\ 0\end{pmatrix} + \begin{pmatrix}0\\ \vec{v}_0\end{pmatrix}$

have Solution $\begin{pmatrix}\vec{r}(t)\\ \vec{v}(t)\end{pmatrix} = \begin{pmatrix}\vec{r}_0 \cos t\\ -\vec{r}_0 \sin t\end{pmatrix} + \begin{pmatrix}\vec{v}_0 \cdot \sin t\\ \vec{v}_0 \cdot \cos t\end{pmatrix}$

$$\begin{pmatrix} \vec{r}(t) \\ \vec{v}(t) \end{pmatrix} = \begin{pmatrix} \vec{r}_0 \cos t + \vec{v}_0 \sin t \\ -\vec{r}_0 \sin t + \vec{v}_0 \cos t \end{pmatrix}$$

$$\Rightarrow \vec{r}(t) = \vec{r}_0 \cos t + \vec{v}_0 \sin t$$

<u>Optional</u>. On the Characteristic.

Find eigenvalue of $\begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}$

$$\begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix} - \lambda I_6 = \begin{pmatrix} -\lambda & & & 1 & & \\ & -\lambda & & & 1 & \\ & & -\lambda & & & 1 \\ -1 & & & -\lambda & & \\ & -1 & & & -\lambda & \\ & & -1 & & & -\lambda \end{pmatrix}$$

$$\det\begin{pmatrix} \cdots \end{pmatrix} = \det\left(\begin{array}{ccc|ccc} -\lambda & & & 1 & & \\ & -\lambda & & & 1 & \\ & & -\lambda & & & 1 \\ \hline 0 & & & -\lambda-\frac{1}{\lambda} & & \\ & 0 & & & -\lambda-\frac{1}{\lambda} & \\ & & 0 & & & -\lambda-\frac{1}{\lambda} \end{array}\right)$$

$$= (-\lambda)^3 \cdot \left(-\lambda - \frac{1}{\lambda}\right)^3 = (\lambda^2+1)^3 = 0$$

$$\Rightarrow \lambda = \pm i.$$

$\lambda_1 = i$.

$$\left(\begin{array}{ccc|ccc} -i & & & 1 & & \\ & -i & & & 1 & \\ & & -i & & & 1 \\ \hline -1 & & & -i & & \\ & -1 & & & -i & \\ & & -1 & & & -i \end{array}\right)\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix} = 0.$$

$$\sim \left(\begin{array}{ccc|ccc} -i & & & 1 & & \\ & -i & & & 1 & \\ & & -i & & & 1 \\ \hline 0 & & & 0 & & \\ & 0 & & & 0 & \\ & & 0 & & & 0 \end{array}\right)\begin{pmatrix} \vdots \end{pmatrix} = 0.$$

eigenvector
$$\begin{pmatrix}1\\0\\0\\i\\0\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\\0\\0\\i\\0\end{pmatrix} \quad \begin{pmatrix}0\\0\\1\\0\\0\\i\end{pmatrix} \sim \begin{pmatrix}\vec{d}_1\\ i\vec{d}_1\end{pmatrix} = \vec{f}_1$$

Similarly

$\lambda_2 = -i$.
$$\begin{pmatrix}1\\0\\0\\-i\\0\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\\0\\0\\-i\\0\end{pmatrix} \quad \begin{pmatrix}0\\0\\1\\0\\0\\-i\end{pmatrix} \sim \begin{pmatrix}\vec{d}_2\\ -i\vec{d}_2\end{pmatrix} = \vec{f}_2$$

~~Now.~~
$$\cancel{\frac{d}{dt}\left[e^{\lambda_i t}\begin{pmatrix}\vec{d}_i\\ i\vec{d}_i\end{pmatrix}\right] = \lambda_i e^{\lambda_i t}\begin{pmatrix}\vec{d}_i\\ i\vec{d}_i\end{pmatrix}}$$
$$\cancel{= e^{\lambda_i t}\lambda_i\begin{pmatrix}\vec{d}_i\\ i\vec{d}_i\end{pmatrix}}$$

Now.
$$\frac{d}{dt}\left(e^{\lambda_i t} f_i\right) = \lambda_i e^{\lambda_i t} f_i = e^{\lambda_i t}\lambda_i f_i$$
$$= e^{\lambda_i t}\cdot\begin{pmatrix}0 & I_3\\ -I_3 & 0\end{pmatrix} f_i$$
$$= \begin{pmatrix}0 & I_3\\ -I_3 & 0\end{pmatrix}\left(e^{\lambda_i t} f_i\right)$$

$$e^{\lambda_1 t} = e^{it} = \cos t + i\sin t \qquad e^{\lambda_1 t} f_1 = \begin{pmatrix}\vec{d}_1\cos t\\ -d_1\sin t\end{pmatrix} + i\begin{pmatrix}d_1\sin t\\ d_1\cos t\end{pmatrix}$$
$$e^{\lambda_2 t} = e^{-it} = \cos t - i\sin t. \qquad e^{\lambda_2 t} f_2 = \begin{pmatrix}d_2\cos t\\ -d_2\sin t\end{pmatrix} + i\begin{pmatrix}-d_2\sin t\\ -d_2\cos t\end{pmatrix}$$

~~Ref.~~ Real & Complex Part gives the characteristics.